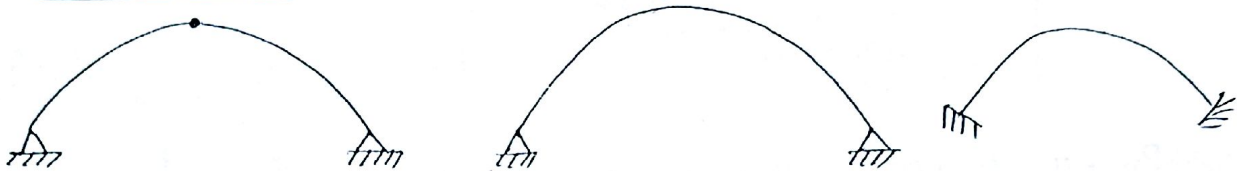


ARCHES

→ 3 Hinged arches - 1 hinge at crown & 2 Hinge at support
(Determinate, stable)

2 Hinged arches - Both hinged at abutments/support.
(Indeterminate, stable)

Fixed arches - Both ends fixed (Stable, Indeterminate)



Advantages of Arches over beams :-

- 1) For same loading and same span, the B.M. in arch section is smaller than BM in beam section at same distance from support, because of horizontal reaction developed at supports of arch.
- 2) For long span, arch section are cheaper due to thinner cross section needed on account of less BM, however for small span, beam may be advantageous.

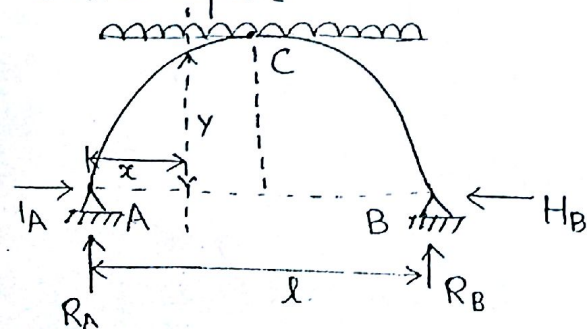
Disadvantages

- 1) Difficult to construct because of curved section.
- 2) For multistory buildings arch provide constraint of height and aesthetically less preferred.

Analysis of 3 Hinged arches

Three hinged arch is determinate and stable.

3 Hinged Parabolic arch subjected to udl over entire span.



$$H_A = H_B$$

$$R_A = R_B = \frac{wl}{2}$$

$$\sum M_C = 0,$$

$$R_A \times \frac{l}{2} - H \times h = \frac{wl}{2} \times \frac{l}{4}$$

$$\frac{wl}{2} \times \frac{l}{2} - H \times h = \frac{wl^2}{8}$$

$$H = \frac{wl^2}{8h}$$

$$y = \frac{4h}{l^2} x(l-x) \quad \leftarrow \text{Parabolic Arch eqn.}$$

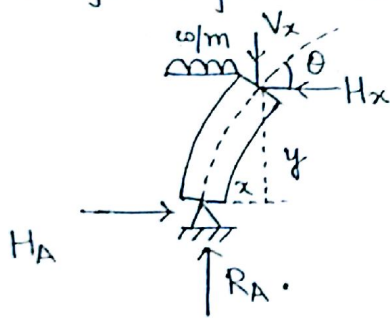
BM at any section x-x,

$$M_x = R_A \cdot x - H_A \cdot y - w x \cdot \frac{x}{2}$$

$$M_x = \frac{wlx}{2} - \frac{wl^2}{8h} \times \frac{4h}{l^2} x(l-x) - \frac{wx^2}{2}$$

$$\boxed{M_x = 0}$$

Parabolic 3 hinged arch with udl on entire span is free from BM.



$$\sum F_x = 0$$

$$H_A - H_x = 0$$

$$H_x = \frac{wl^2}{8h}$$

$$R_A - wx - V_x = 0$$

$$\boxed{V_x = \frac{wl}{2} - wx}$$

Shear force in arch is also called radial shear which is tangential to section x-x,

$$\boxed{S_x = V_x \cos \theta - H_x \sin \theta}$$

Normal thrust @ axial thrust at x-x,

$$\boxed{N_x = V_x \sin \theta + H_x \cos \theta}$$

$$S_x = V_x \cos \theta - H_x \sin \theta$$

$$y = \frac{4h}{l^2} x(l-x)$$

$$\frac{dy}{dx} = \frac{4h}{l^2} (l-2x)$$

$$\tan \theta = \frac{4h}{l^2} (l-2x)$$

$$S_x = \left(\frac{wl}{2} - wx \right) \cos \theta - \frac{wl^2}{8h} \sin \theta$$

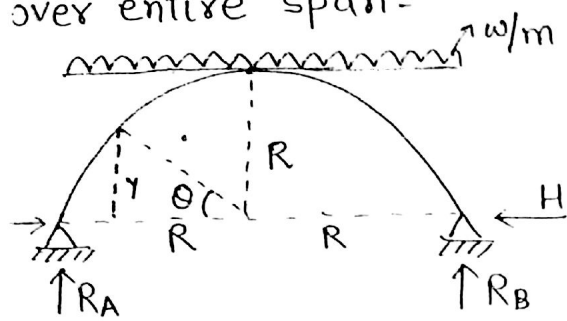
$$= \cos \theta \left[\frac{wl}{2} - wx - \frac{wl^2}{8h} \tan \theta \right]$$

$$= \cos \theta \left[\frac{wl}{2} - wx - \frac{wl}{2} + wx \right]$$

$$\boxed{S_x = 0}$$

It means, 3 hinged parabolic arch with udl is free from shear force and BM and is subjected to only axial thrust @ normal thrust.

Three hinged semi-circular arch subjected to udl over entire span-



$$R_A = R_B = \frac{1}{2} \times w \times 2R = wR$$

$$\sum M_c = 0,$$

$$R_A \times R - H \times R - wR \times \frac{R}{2} = 0$$

$$H = wR - \frac{wR}{2}$$

$$\boxed{H = \frac{wR}{2}}$$

→ Horizontal thrust is half of vertical reaction in three hinged semi-circular arch if loading is udl over entire span.

BM. at x-x,

$$M_x = R_A \cdot x - Hy - w x \cdot \frac{x}{2}$$

$$= wR x - \frac{wR}{2} y - \frac{w x^2}{2}$$

$$= wR^2 (1 - \cos \theta) - \frac{wR}{2} \times R \sin \theta - \frac{w}{2} [R(1 - \cos \theta)]^2$$

$$= wR^2 \left[(1 - \cos \theta) \left(\frac{1 + \cos \theta}{2} \right) - \frac{\sin \theta}{2} \right]$$

$$= \frac{wR^2}{2} [\sin^2 \theta - \sin \theta]$$

$$= -\frac{wR^2}{2} [\sin \theta - \sin^2 \theta]$$

$$\text{since, } \sin \theta \geq \sin^2 \theta$$

$$\boxed{M_x \leq 0} \quad (\text{Hogging})$$

M_x to be maximum,

$$\frac{dM}{d\theta} = 0$$

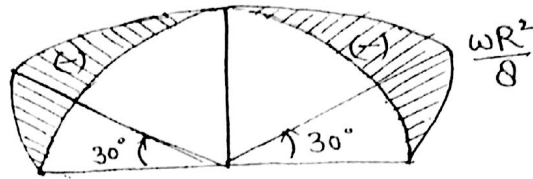
$$-\frac{wR^2}{2} [\cos \theta - 2 \sin \theta \cdot \cos \theta] = 0$$

$$\boxed{\theta = 30^\circ}$$

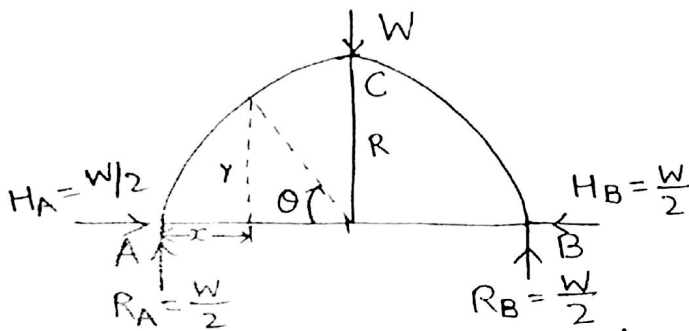
M_{\max} occurs at $\theta = 30^\circ$,

$$\left\{ M_{\max} = -\frac{wR^2}{2} \left[\frac{1}{2} - \frac{1}{4} \right] = -\frac{wR^2}{8} \right\}$$

At $\theta = 30^\circ$ with horizontal, $M_{\max} = -\frac{wR^2}{8}$



For three hinged semi-circular arch subjected to concentrated load at crown W . Draw shear force and B.M. diagram.



$$R_A = R_B = W/2$$

$$H_A \times R = R_A \times R$$

$$H_A = R_A = W/2$$

$$y = R \sin \theta$$

$$x = R(1 - \cos \theta)$$

$$\sum M_C = 0,$$

$$M_x = R_A \cdot x - H_A \cdot y$$

$$= \frac{W}{2} \cdot x - \frac{W}{2} \cdot R \sin \theta$$

$$= \frac{WR}{2} [1 - \cos \theta - \sin \theta]$$

For M_x to be max,

$$\frac{\partial M}{\partial \theta} = 0 \Rightarrow \theta = 45^\circ$$

$$M_{x,\max} = -\frac{wR}{2} (\sqrt{2} - 1)$$

Shear force at section x , $= \frac{W}{2} \sin \theta - \frac{W}{2} \cos \theta$

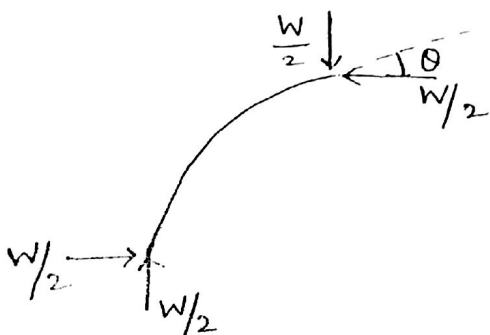
$$V_x = \frac{W}{2} (\sin \theta - \cos \theta)$$

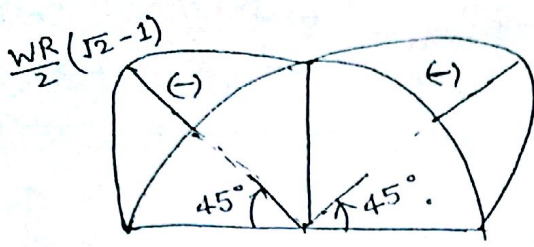
at $\theta = 0$,

$$V_x = -W/2$$

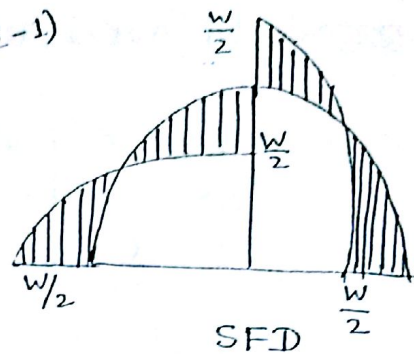
at $\theta = \pi/2$

$$V_x = +W/2$$



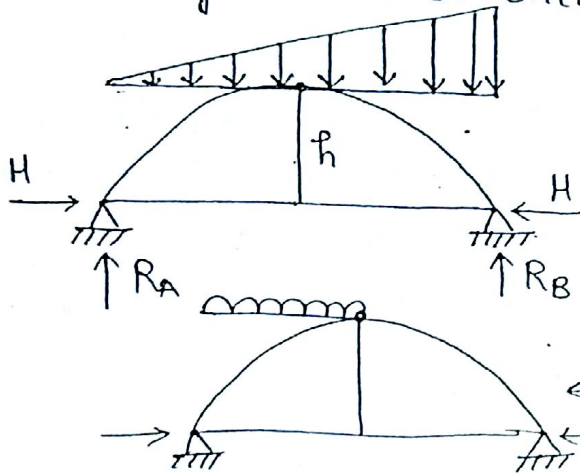


BMD



SFD

Question- Find horizontal thrust in 3 hinged parabolic arch by uvl over entire span.



(a). $\frac{wl^2}{8h}$

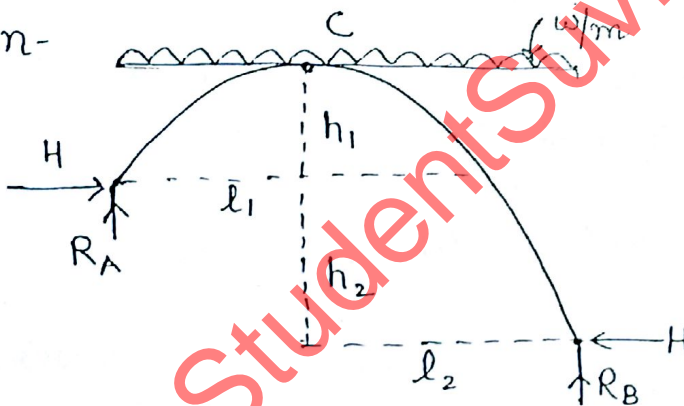
(b). $\frac{wl^2}{16h}$

(c). $\frac{wl^2}{24h}$

(d). $\frac{wl^2}{0}$

← similar case → $H = \frac{wl^2}{16h}$

Question-



$y \propto x^2$

$\frac{x^2}{y} = \text{constt.}$

$\frac{x}{\sqrt{y}} = \text{constt.}$

$\boxed{\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}}}$

$R_A + R_B = wl$

$\Sigma M_C = 0$ $R_A \times l_1 - H h_1 - w l_1 \times \frac{l_1}{2} = 0$

$\left\{ H = \frac{R_A l_1 - w l_1^2}{h_1} \right\}$

$\frac{l_1}{l_2} = \sqrt{h_1/h_2} \Rightarrow$

$\frac{l_1 + l_2}{l_2} = \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_2}}$

$\boxed{l_2 = \frac{l \sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}}$
 $\boxed{l_1 = \frac{l \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}}$

Temperature effect on three hinge arches-

Three hinged arch is a determinate structure. If arch is initially unloaded (neglected self weight), then due to rise of temperature there will be no horizontal and vertical reaction introduced. In parabolic three hinged arch the increase in vertical rise will be Δh which is given by,

$$\Delta h = \left[\frac{l^2 + 4h^2}{4h} \right] \alpha t$$

Δh is called free expansion of vertical hinge.

If before temperature change, arch is loaded say udl over entire span then horizontal thrust H due to loading,

$$H = \frac{wl^2}{8h}$$

on heating, vertical rise h increases. Hence, horizontal thrust will reduce. The change in horizontal thrust is given as,

$$dH = \frac{wl^2}{8} \left[-\frac{1}{h^2} \cdot dh \right]$$

% change in H ,

$$\left\{ \frac{dH}{H} \times 100 = -\frac{dh}{h} \times 100 \right\}$$

NOTE- Two hinged arch is an indeterminate structure. If two hinged arch is unloaded and there is no horizontal thrust then on heating horizontal thrust is produced and if it is loaded and has horizontal thrust before heating then after heating horizontal thrust will increase.